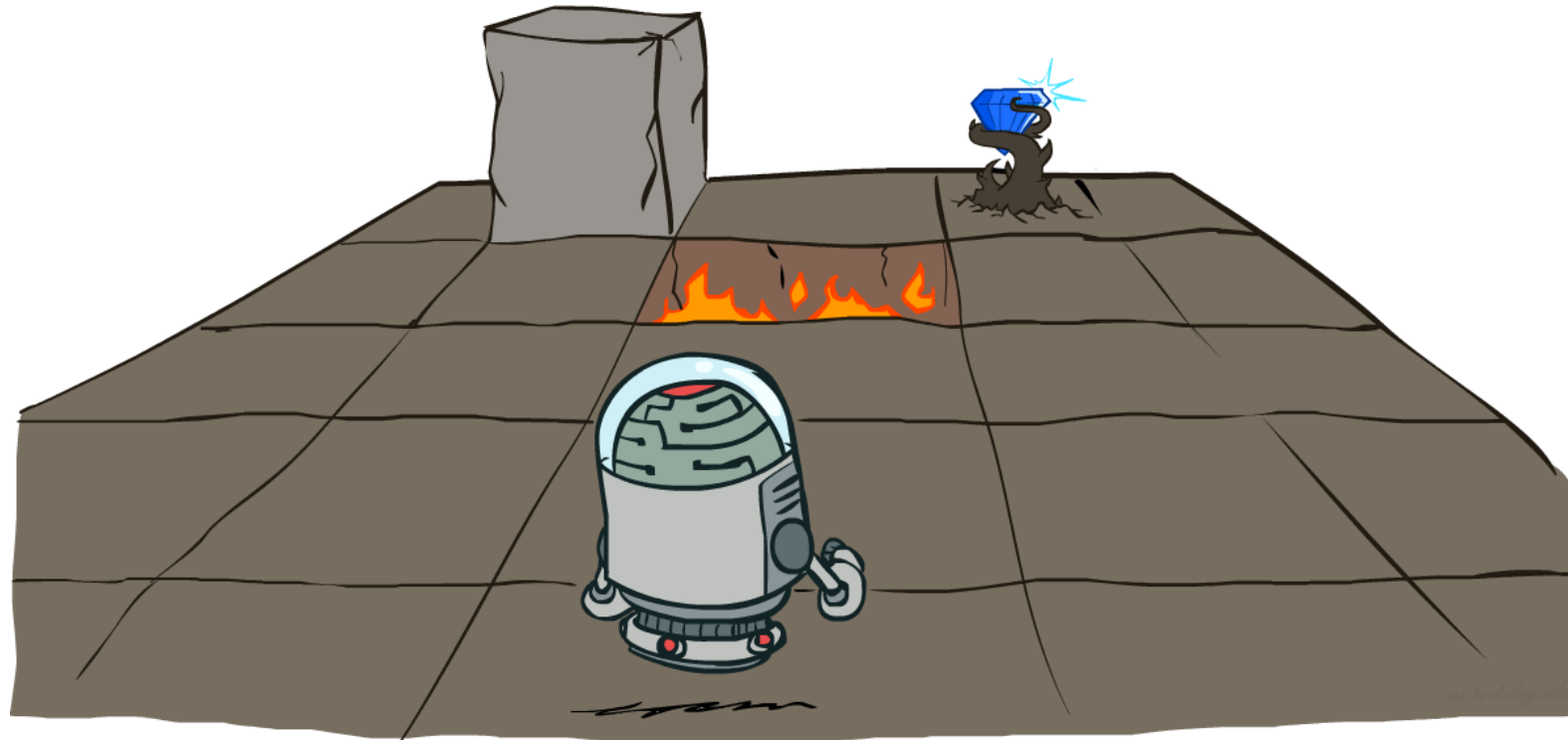


CS 188: Artificial Intelligence

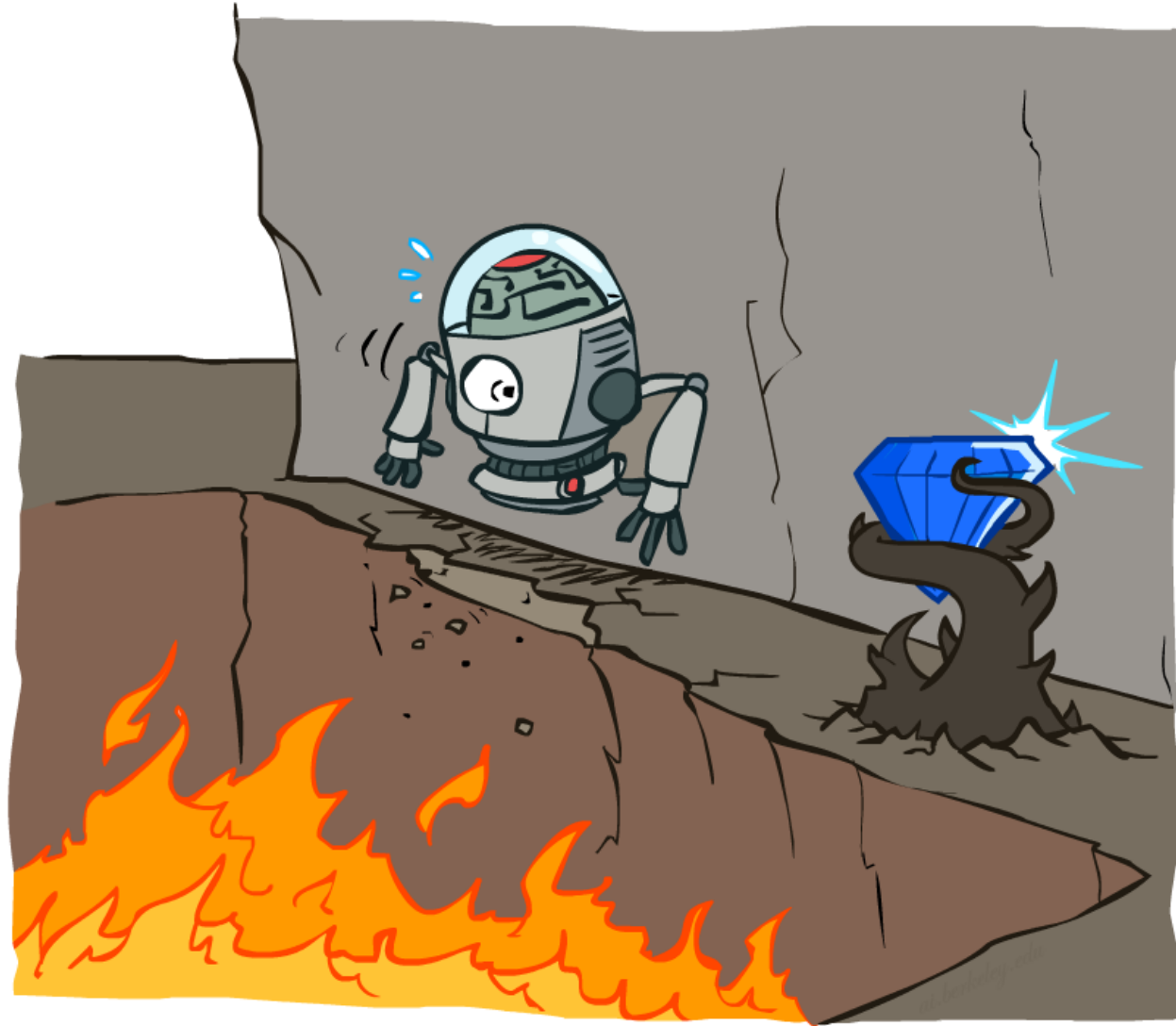
Markov Decision Processes



Instructors: Dan Klein and Pieter Abbeel

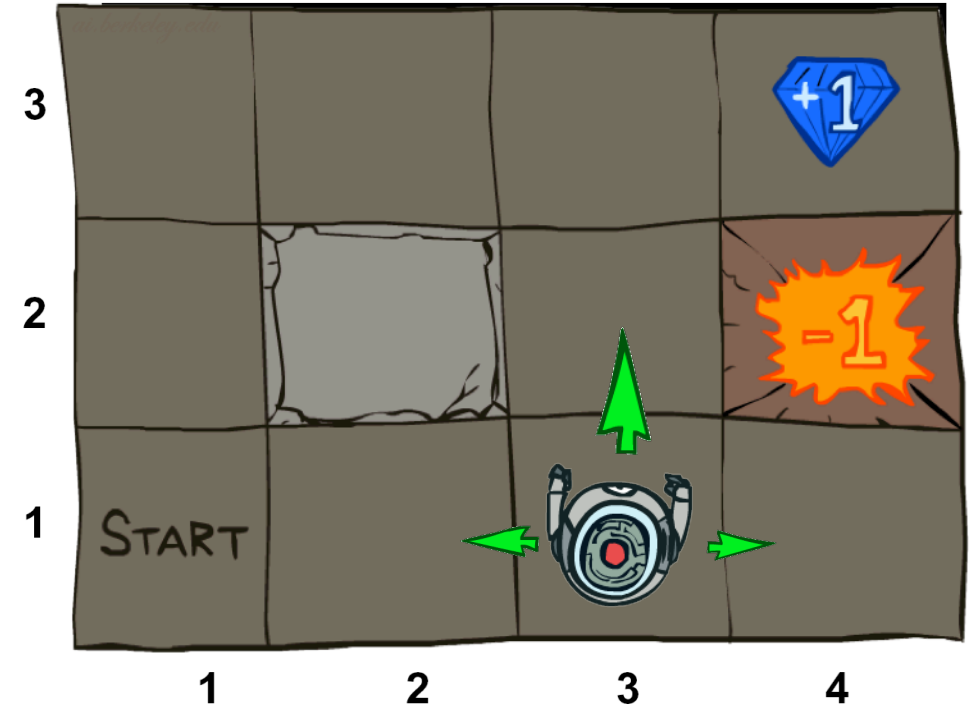
University of California, Berkeley

Non-Deterministic Search



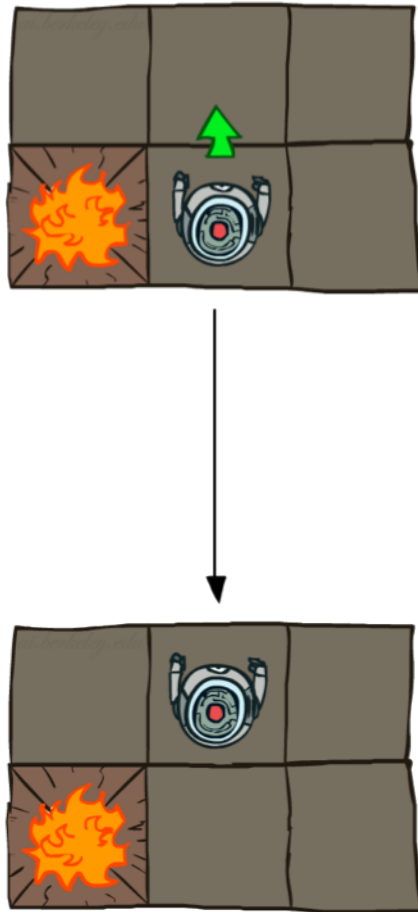
Example: Grid World

- A maze-like problem
 - The agent lives in a grid
 - Walls block the agent's path
- Noisy movement: actions do not always go as planned
 - 80% of the time, the action North takes the agent North (if there is no wall there)
 - 10% of the time, North takes the agent West; 10% East
 - If there is a wall in the direction the agent would have been taken, the agent stays put
- The agent receives rewards each time step
 - Small "living" reward each step (can be negative)
 - Big rewards come at the end (good or bad)
- Goal: maximize sum of rewards

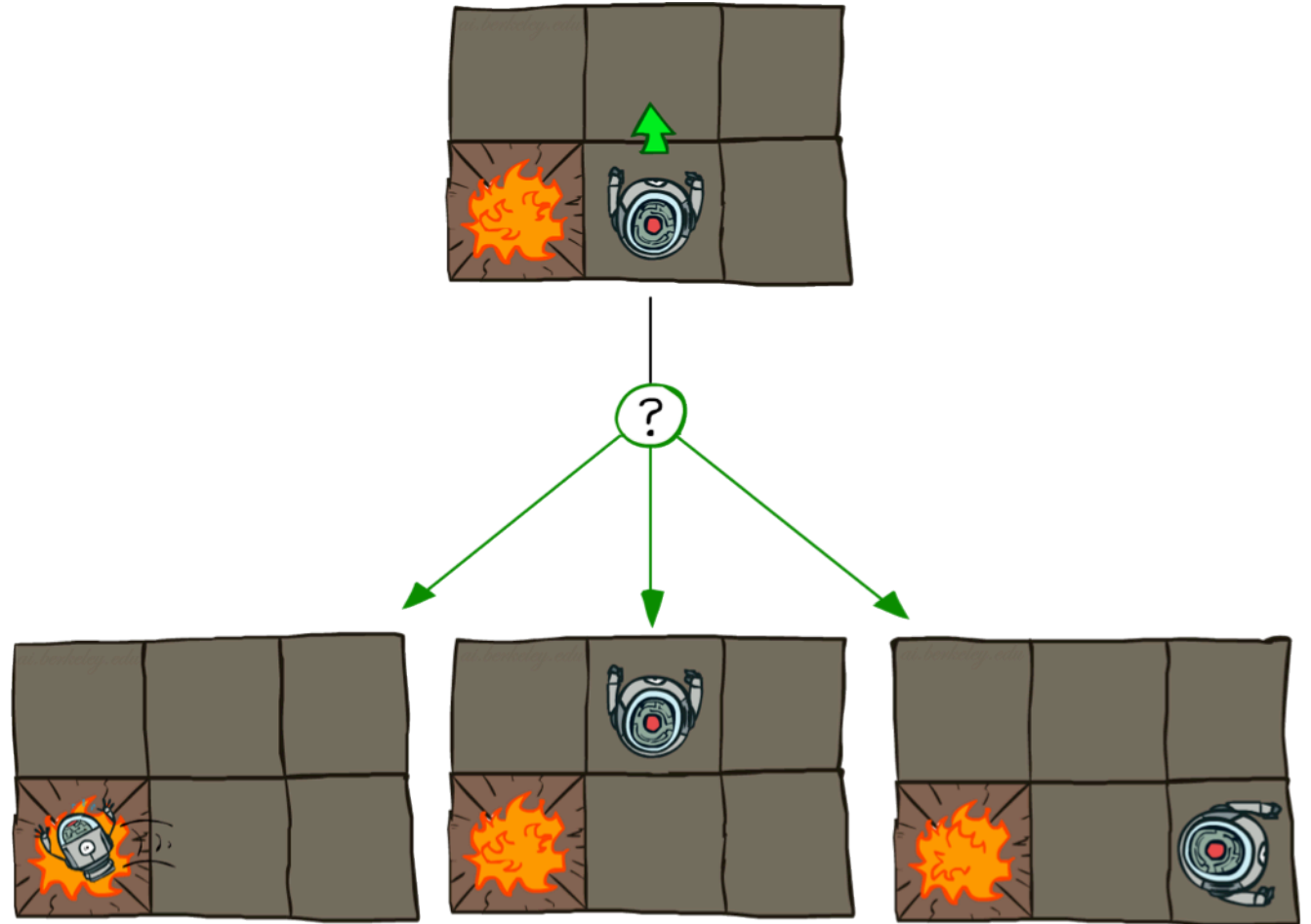


Grid World Actions

Deterministic Grid World

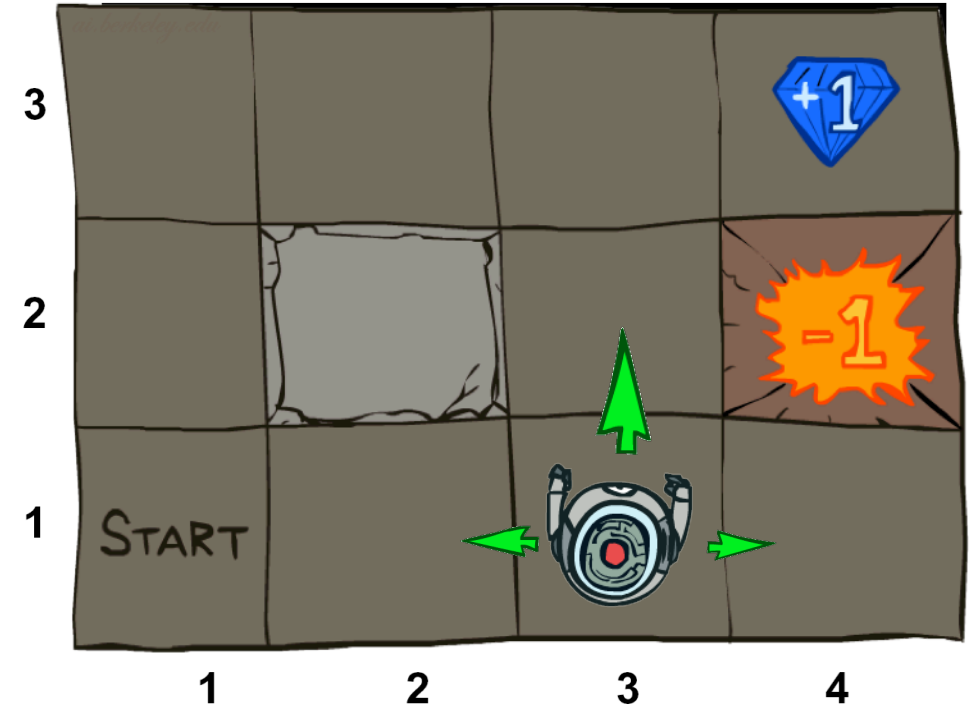


Stochastic Grid World



Markov Decision Processes

- An MDP is defined by:
 - A **set of states** $s \in S$
 - A **set of actions** $a \in A$
 - A **transition function** $T(s, a, s')$
 - Probability that a from s leads to s' , i.e., $P(s' | s, a)$
 - Also called the model or the dynamics
 - A **reward function** $R(s, a, s')$
 - Sometimes just $R(s)$ or $R(s')$
 - A **start state**
 - Maybe a **terminal state**
- MDPs are non-deterministic search problems
 - One way to solve them is with expectimax search
 - We'll have a new tool soon



Video of Demo Gridworld Manual Intro



What is Markov about MDPs?

- “Markov” generally means that given the present state, the future and the past are independent
- For Markov decision processes, “Markov” means action outcomes depend only on the current state

$$P(S_{t+1} = s' | S_t = s_t, A_t = a_t, S_{t-1} = s_{t-1}, A_{t-1}, \dots, S_0 = s_0)$$

=

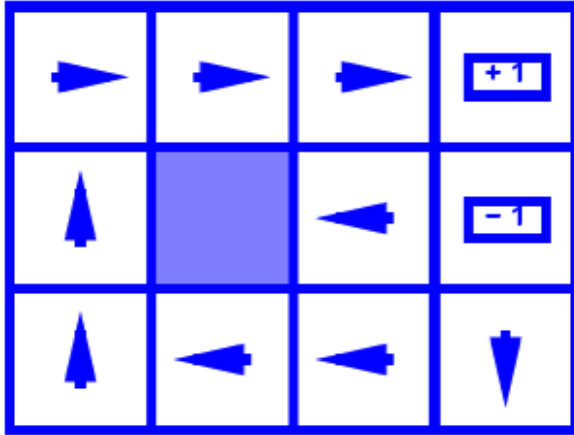
$$P(S_{t+1} = s' | S_t = s_t, A_t = a_t)$$

- This is just like search, where the successor function could only depend on the current state (not the history)

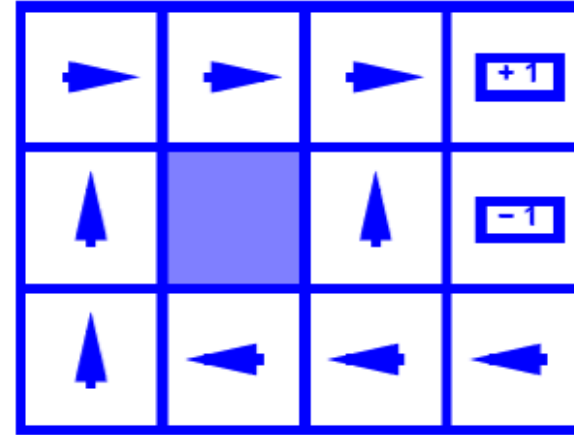


Andrey Markov
(1856-1922)

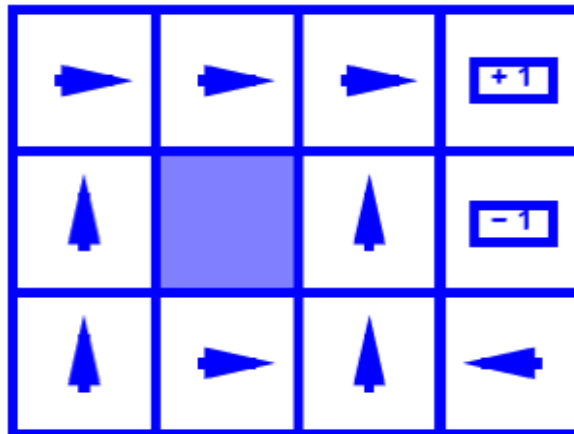
Optimal Policies



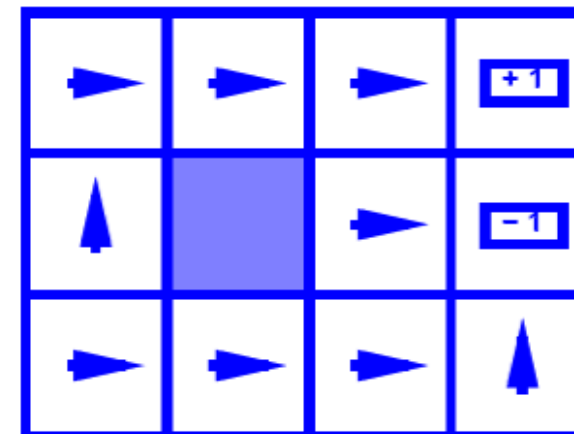
$R(s) = -0.01$



$R(s) = -0.03$

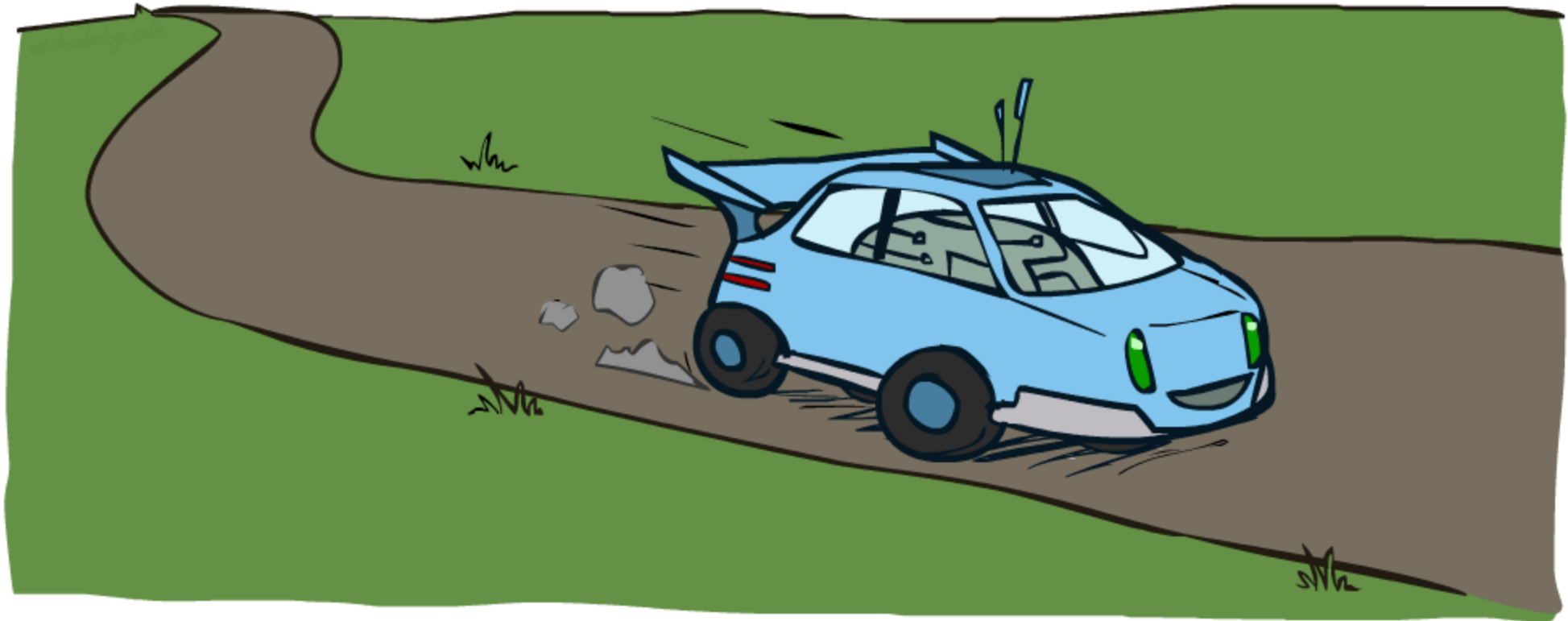


$R(s) = -0.4$



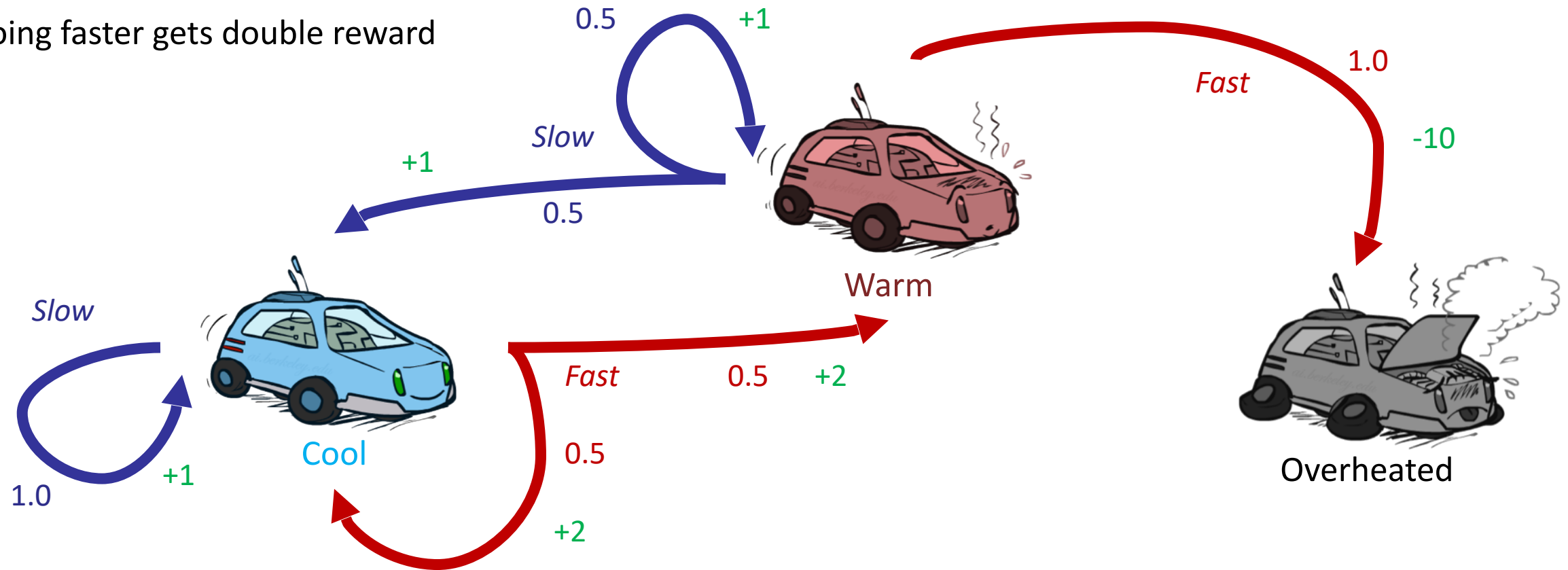
$R(s) = -2.0$

Example: Racing



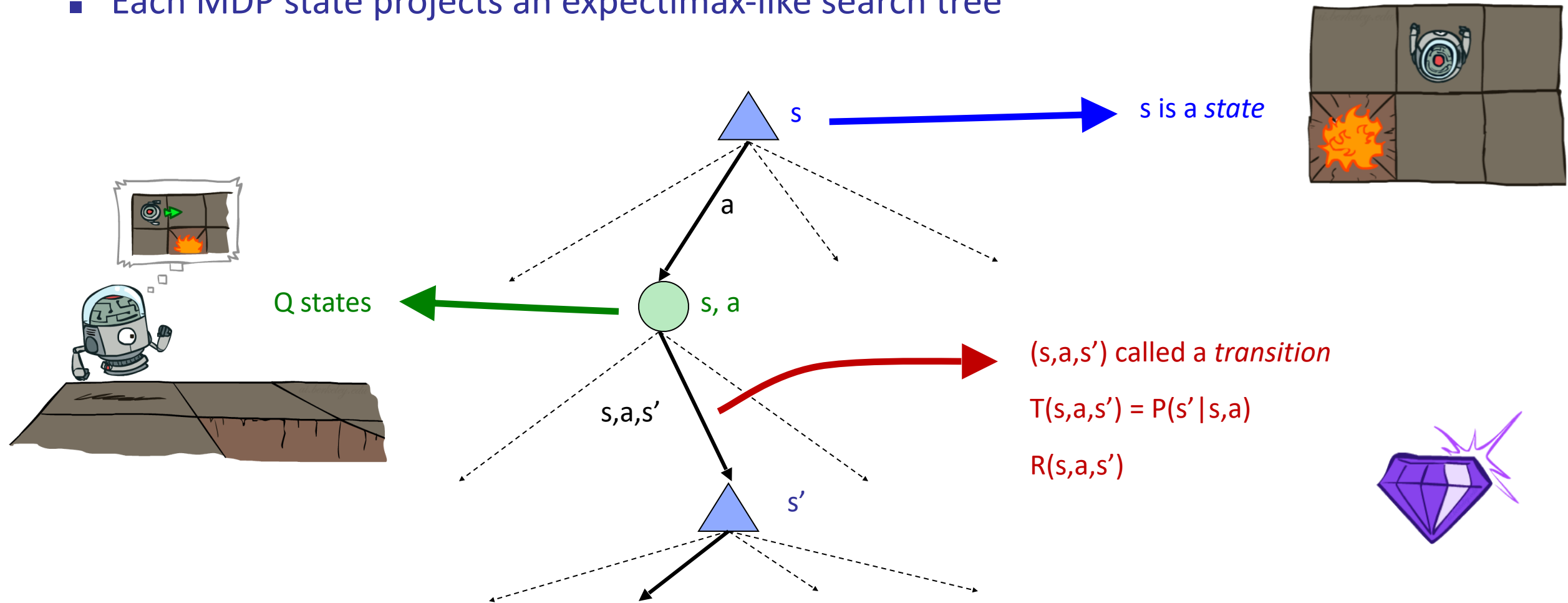
Example: Racing

- A robot car wants to travel far, quickly
- Three states: **Cool**, **Warm**, Overheated
- Two actions: **Slow**, **Fast**
- Going faster gets double reward

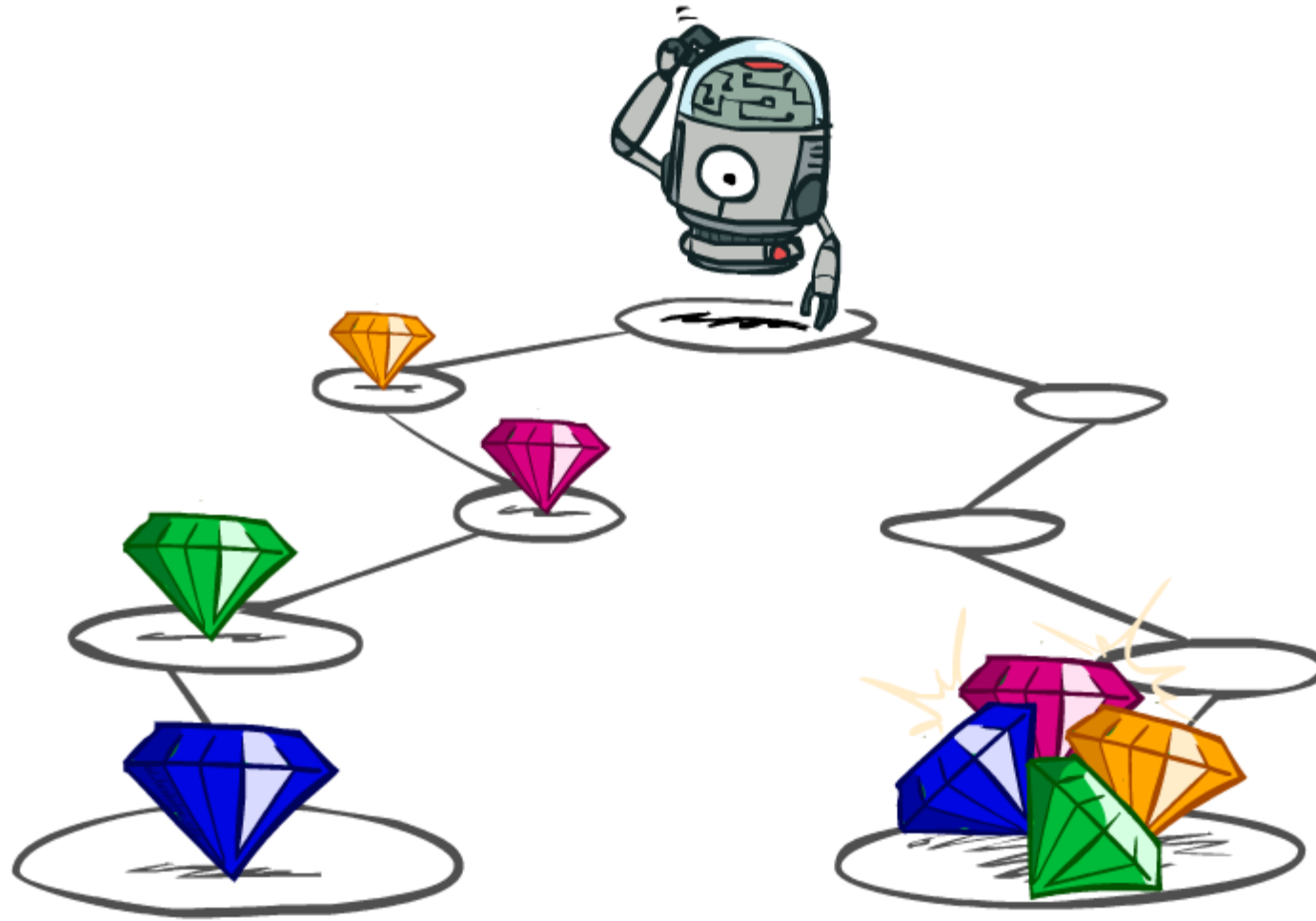


MDP Search Trees

- Each MDP state projects an expectimax-like search tree

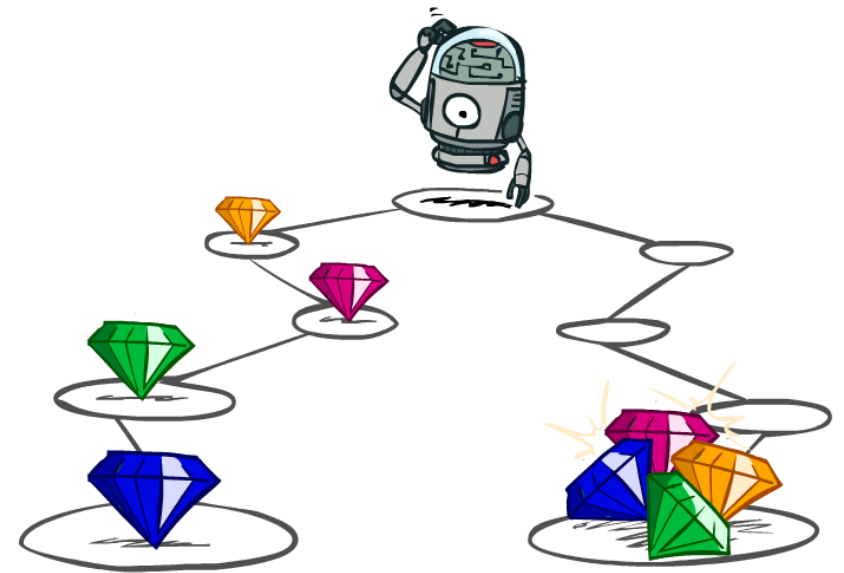


Utilities of Sequences



Utilities of Sequences

- What preferences should an agent have over reward sequences?
- More or less? $[1, 2, 2]$ or $[2, 3, 4]$
- Now or later? $[0, 0, 1]$ or $[1, 0, 0]$



Discounting

- It's reasonable to maximize the sum of rewards
- It's also reasonable to prefer rewards now to rewards later
- One solution: values of rewards decay exponentially



1

Worth Now



γ

Worth Next Step



γ^2

Worth In Two Steps

Discounting

- How to discount?

- Each time we descend a level, we multiply in the discount once



1

- Why discount?

- Sooner rewards probably do have higher utility than later rewards
- Also helps our algorithms converge



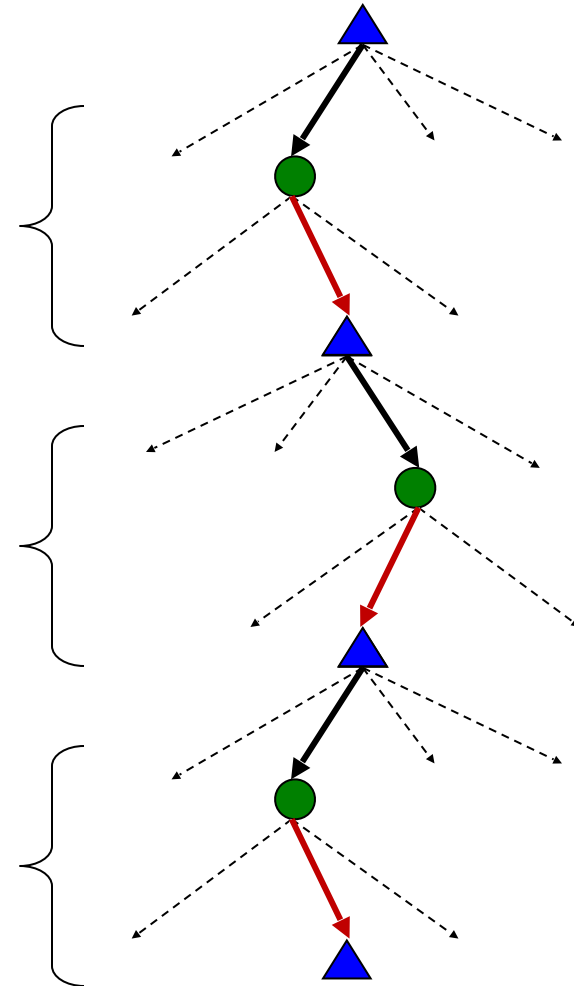
γ

- Example: discount of 0.5

- $U([1,2,3]) = 1*1 + 0.5*2 + 0.25*3$
- $U([1,2,3]) < U([3,2,1])$



γ^2



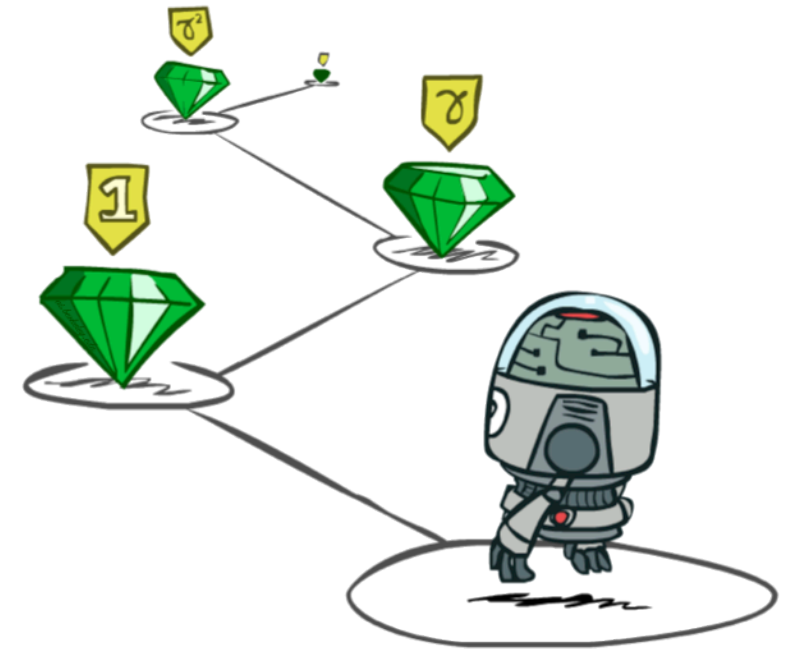
Stationary Preferences

- Theorem: if we assume **stationary preferences**:

$$[a_1, a_2, \dots] \succ [b_1, b_2, \dots]$$



$$[r, a_1, a_2, \dots] \succ [r, b_1, b_2, \dots]$$



- Then: there are only two ways to define utilities

- Additive utility:

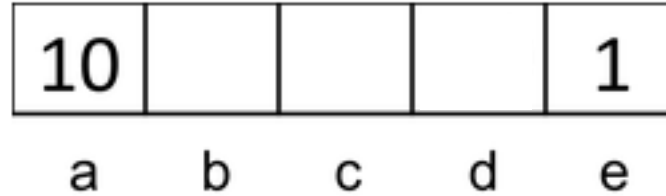
$$U([r_0, r_1, r_2, \dots]) = r_0 + r_1 + r_2 + \dots$$

- Discounted utility:

$$U([r_0, r_1, r_2, \dots]) = r_0 + \gamma r_1 + \gamma^2 r_2 \dots$$

Quiz: Discounting

- Given:



- Actions: East, West, and Exit (only available in exit states a, e)
- Transitions: deterministic

- Quiz 1: For $\gamma = 1$, what is the optimal policy?



- Quiz 2: For $\gamma = 0.1$, what is the optimal policy?



- Quiz 3: For which γ are West and East equally good when in state d?

Infinite Utilities?!

- Problem: What if the game lasts forever? Do we get infinite rewards?

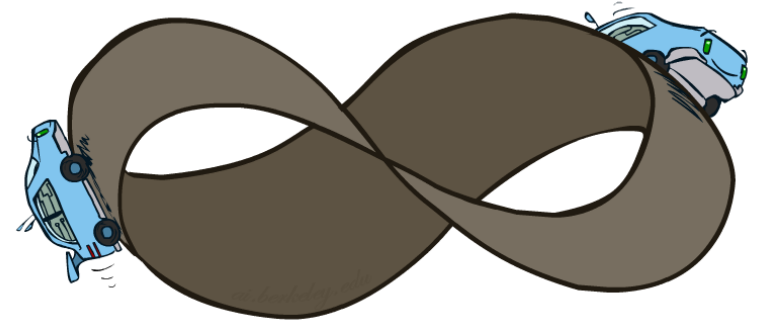
- Solutions:

- Finite horizon: (similar to depth-limited search)
 - Terminate episodes after a fixed T steps (e.g. life)
 - Gives nonstationary policies (π depends on time left)

- Discounting: use $0 < \gamma < 1$

$$U([r_0, \dots, r_\infty]) = \sum_{t=0}^{\infty} \gamma^t r_t \leq R_{\max} / (1 - \gamma)$$

- Smaller γ means smaller “horizon” – shorter term focus
- Absorbing state: guarantee that for every policy, a terminal state will eventually be reached (like “overheated” for racing)



Recap: Defining MDPs

- Markov decision processes:
 - Set of states S
 - Start state s_0
 - Set of actions A
 - Transitions $P(s' | s, a)$ (or $T(s, a, s')$)
 - Rewards $R(s, a, s')$ (and discount γ)

- MDP quantities so far:
 - Policy = Choice of action for each state
 - Utility = sum of (discounted) rewards

